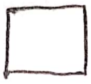
Loop Analysis (Mesh Analysis) (DC Excitation)

Note: Procedure to solve problems on loop analysis (Both DC & AC excitations)

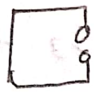
i) Identify the number of meshes

ii) Assign clockwise direction for mesh currents

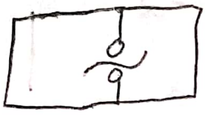
iii) To identify essential, nonessential and supermesh; in place of current source put a open and the remaining elements as lines.



This is a essential mesh and we write KVL equation (for closed path)



This is a nonessential mesh and we write KCL equation (for current source)



This is supermesh and we write KVL & KCL equations.
Two conditions has to be met to form a super mesh
i) Meshes has to merge
ii) The outer path should be a closed path.

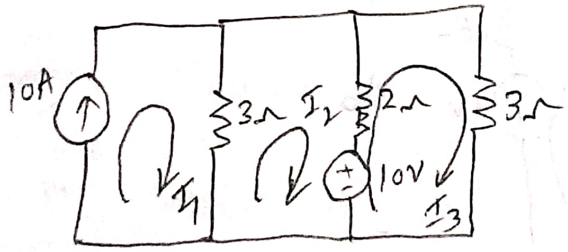
(only when there is a current source in a circuit, we come across nonessential & supermesh)

iv) Equations are sorted out & by making use of calculator, solve the simultaneous equations for unknown currents $I_1, I_2 \ldots$

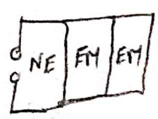
v) Voltage across elements & power dissipated can be obtained by using mesh currents.

Prob: Write the mesh equations for the circuit shown in figure and determine mesh currents using mesh analysis.

Sol:



Rough work (procedure iii)



KCL equation for non-essential mesh 1 : $I_1 = +10$

KVL equation for essential mesh 2 : $3(I_2 - I_1) + 2(I_2 - I_3) + 10 = 0$

KVL equation for essential mesh 3 : $-10 + 2(I_3 - I_2) + 3I_3 = 0$

Sorting the above equations : $(1)I_1 + (0)I_2 + (0)I_3 = 10$

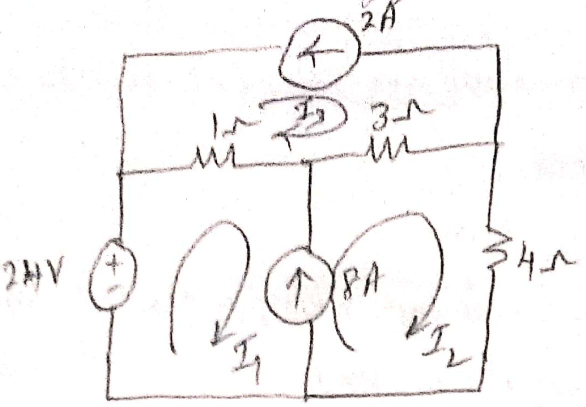
$(-3)I_1 + (5)I_2 + (-2)I_3 = -10$

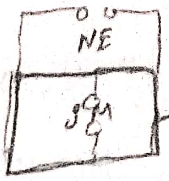
$(0)I_1 + (-2)I_2 + (5)I_3 = 10$

Using calculator, $I_1 = 10A$, $I_2 = 5.714A$, $I_3 = 4.285A$

Prob: For the Circuit shown in the figure, solve for mesh currents

Sol:





→ outer path of SM is closed path (To apply KVL)

KCL equation to nonessential mesh 3 : $I_3 = -2$

KCL equation to supermesh 1 & 2 ; $I_2 - I_1 = 8$

KVL equation to supermesh 1 & 2 ; $-24 + 1(I_1 - I_3) + 3(I_2 - I_3) + 4I_2 = 0$

$$(0)I_1 + (0)I_2 + (1)I_3 = -2$$

$$(-1)I_1 + (1)I_2 + (0)I_3 = 8$$

$$(1)I_1 + (7)I_2 + (-4)I_3 = 24$$

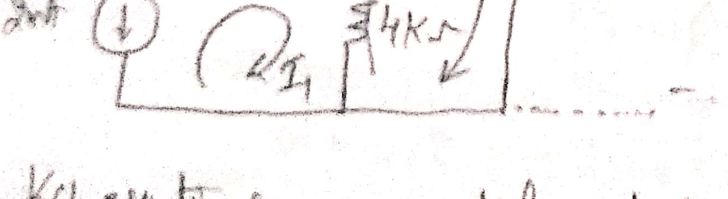
$$I_1 = -5A$$

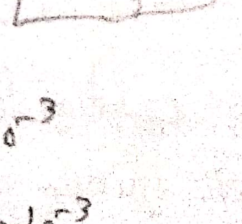
$$I_2 = 3A$$

$$I_3 = -2A$$

Prob: Find V_o in the Circuit shown in the figure using mesh analysis.

Sol:





KCL equation for non-essential mesh 1 : $I_1 = -2 \times 10^{-3}$

KCL equation for non-essential mesh 2 : $I_2 = +4 \times 10^{-3}$

KVL equation for essential mesh 3 : $4K(I_3 - I_1) + 2K(I_3 - I_2) + 6KI_3 - 3$

$$(1)I_1 + (0)I_2 + (0)I_3 = -2\times10^{-3}$$

$$(0)I_1 + (1)I_2 + (0)I_3 = 4\times10^{-3}$$

$$(-4K)I_1 + (-2K)I_2 + (12K)I_3 = 3$$

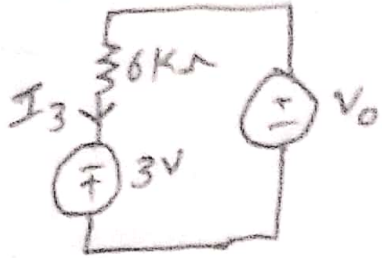
While using calculator in place of K use 10^3 say $-4K$ as -4×10^3

$$I_1 = -2\,mA$$

$$I_2 = 4\,mA$$

$$I_3 = 2.5\times10^{-4}A = 0.25\,mA$$

To find V_0:



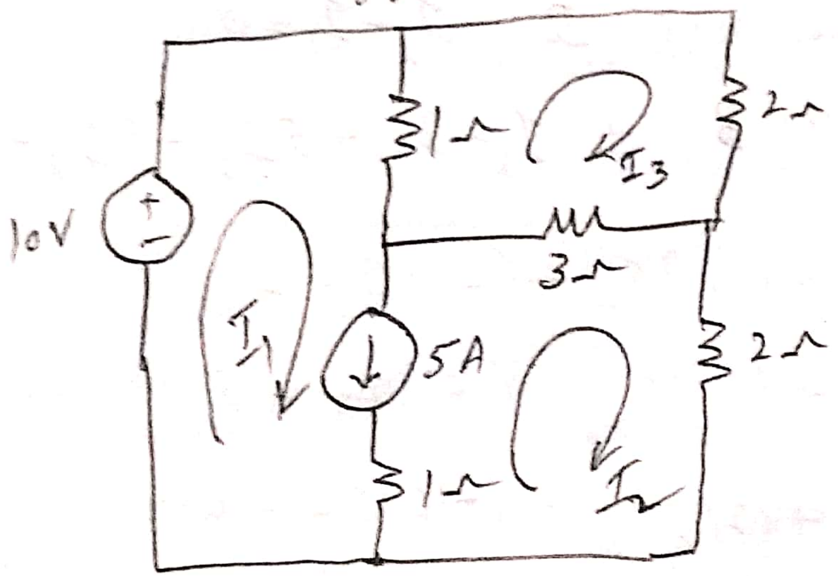
KVL: $+3 - 6K I_3 + V_0 = 0$

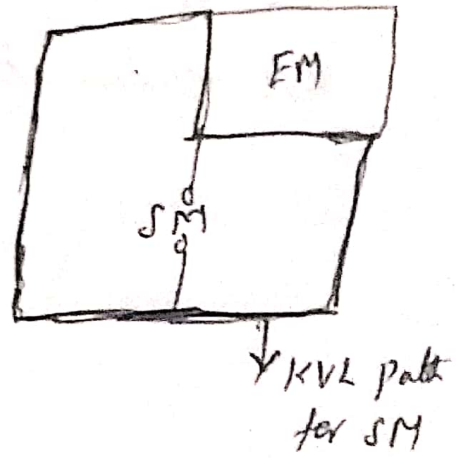
$$V_0 = 6K I_3 - 3$$

$$V_0 = 6\times10^{3}\times0.25\times10^{-3} - 3 = -1$$

Q.3: For the network shown in figure find the mesh currents I_1, I_2 & I_3.

Sol:





KCL equation for super mesh 1 & 2 : $I_1 - I_2 = 5$

KVL equation for essential mesh 3 : $1(I_3 - I_1) + 2I_3 + 3(I_3 - I_2) = 0$

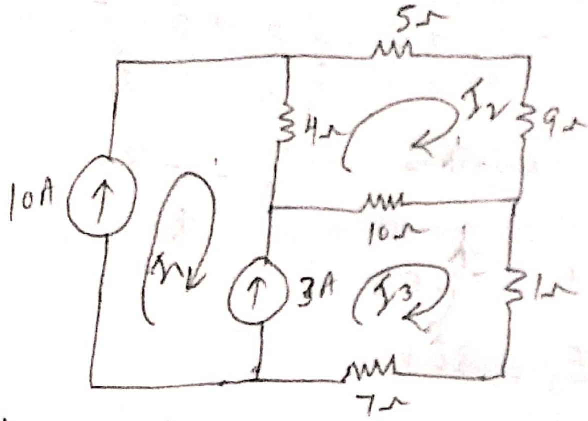
KVL equation for super mesh 1 & 2 : $-10 + 1(I_1 - I_3) + 3(I_2 - I_3) + 2I_2 = 0$

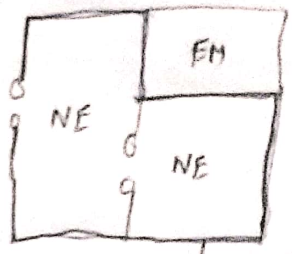
$$(1)I_1 + (-1)I_2 + (0)I_3 = 5$$
$$(-1)I_1 + (-3)I_2 + (6)I_3 = 0$$
$$(1)I_1 + (5)I_2 + (-4)I_3 = 10$$

$$I_1 = 7.5A$$
$$I_2 = 2.5A$$
$$I_3 = 2.5A$$

Prob: Determine the currents $i_1, i_2 \& i_3$ in the circuit shown in the figure using mesh current method.

Sol:





Not a supermesh no closed path eventhough the meshes are merging (1&3)

KCL equation to non-essential mesh 1: $I_1 = 10$

KCL equation to non-essential mesh 3: $I_2 - I_1 = 3$

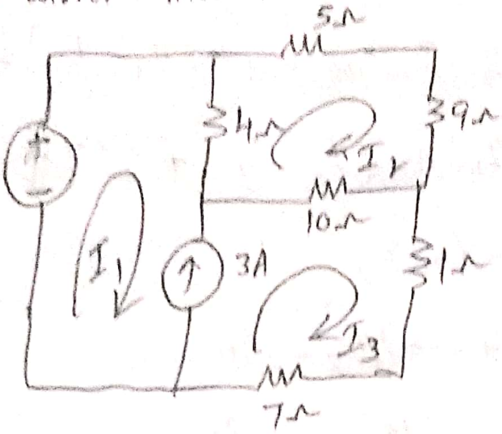
KVL equation to essential mesh 2: $4(I_2 - I_1) + 5I_2 + 9I_2 + 10(I_2 - I_3) = 0$

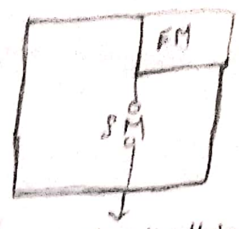
$$(1)I_1 + (0)I_2 + (0)I_3 = 10$$
$$(-1)I_1 + (1)I_2 + (0)I_3 = 3$$
$$(-4)I_1 + (28)I_2 + (-10)I_3 = 0$$

$$I_1 = 10A$$
$$I_2 = 13A$$
$$I_3 = 32.4A$$

Prob: Determine the Currents i_1, i_2 & i_3 in the circuit shown in the figure using mesh current method.

Sol:





closed path to apply KVL

KCL equation to supermesh 1&3 : $I_3 - I_1 = 3$

KVL equation to Supermesh 1&3 : $-10 + 4(I_1 - I_2) + 10(I_3 - I_2) + 1I_3 + 7I_3 = 0$

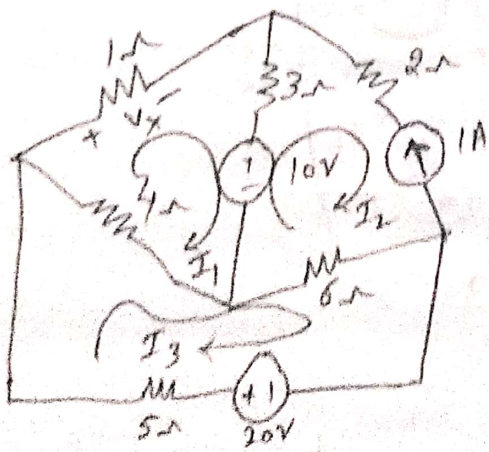
KVL equation to essential mesh 2 : $4(I_2 - I_1) + 5I_2 + 9I_2 + 10(I_2 - I_3) = 0$

$$(-1)I_1 + (0)I_2 + (1)I_3 = 3$$
$$(4)I_1 + (-14)I_2 + (18)I_3 = 10$$
$$(-4)I_1 + (28)I_2 + (-10)I_3 = 0$$

$$I_1 = -1.933\,A$$
$$I_2 = 0.104\,A$$
$$I_3 = 1.066\,A$$

Prob: Using mesh current analysis determine V_x and power supplied by 10V source of the network shown in figure.

Sol:



KCL to nonessential mesh 2 : $I_2 = -1$

KVL to essential mesh 1 : $1I_1 + 3(I_1 - I_2) + 10 + 4(I_1 - I_3) = 0$

KVL to essential mesh 3 : $4(I_3 - I_1) + 6(I_3 - I_2) - 20 + 5I_3 = 0$

$$(0)I_1 + (1)I_2 + (0)I_3 = -1$$

$$(8)I_1 + (-3)I_2 + (-4)I_3 = -10$$

$$(-4)I_1 + (-6)I_2 + (15)I_3 = 20$$

$$I_1 = -1.336\,A$$

$$I_2 = -1\,A$$

$$I_3 = 0.576\,A$$

$$V_x = +1I_1 = 1 \times -1.336 = -1.336\,V$$

$$P_{absorbed\,(10V)} = +VI$$

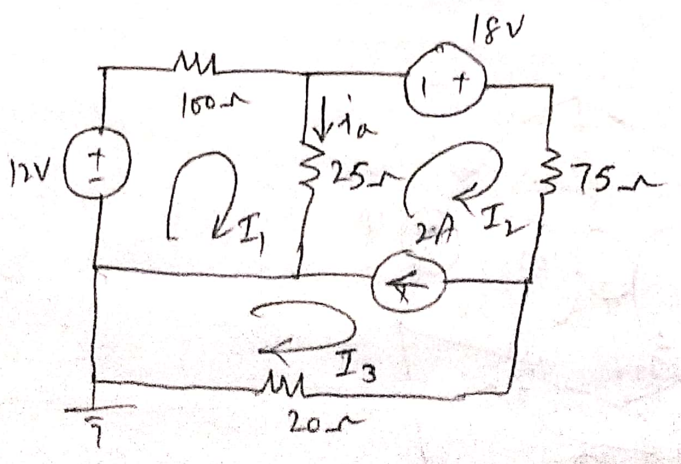
$$= +(V)(I_1 - I_2)$$

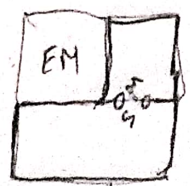
$$= +(10)(-1.336 + 1)$$

$$= -3.36\,W$$

$$P_{delivered\,(10V)} = +3.36\,W$$

Prob: Find the current i_a in the circuit shown in figure using mesh analysis.





KVL path for super mesh

Note: If any of the current is given as anticlockwise, make it clockwise while solving the problem. Write the current of opposite sign in the answer.

KCL to Supermesh 2 & 3: $I_2 - I_3 = 2$

KVL to super mesh 2 & 3; $25(I_2 - I_1) - 18 + 75 I_2 + 20 I_3 = 0$

KVL to essential mesh 1: $-12 + 100 I_1 + 25(I_1 - I_2) = 0$

$$(0)I_1 + (1)I_2 + (-1)I_3 = 2$$

$$(-25)I_1 + (100)I_2 + (20)I_3 = 18$$

$$(125)I_1 + (-25)I_2 + (0)I_3 = 12$$

$$I_1 = 0.20A$$

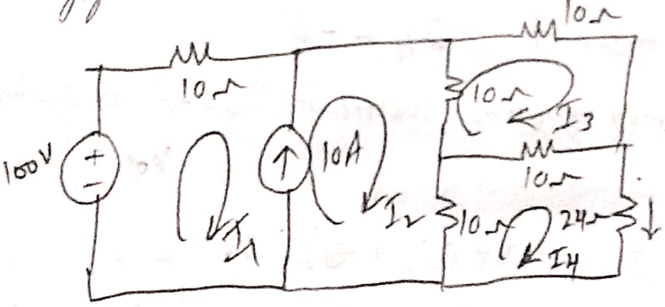
$$I_2 = 0.52A$$

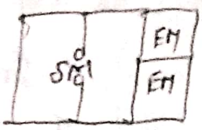
$$I_3 = -1.47A$$

$$i_a = I_1 - I_2 = 0.20 - 0.52 = -0.32A$$

Prob: Find the current in the 24Ω resistance using mesh analysis for the circuit shown in figure.

Sol:





KCL equation for supermesh 1 & 2: $I_2 - I_1 = 10 \Rightarrow I_2 = 10 + I_1$

Since we get four equations, make it as three equations by substituting $I_2 = 10 + I_1$ in the equations below such that the unknowns will be I_1, I_3 & I_4

KVL equation for super mesh 1 & 2: $-100 + 10I_1 + 10(\underbrace{10 + I_1}_{I_2} - I_3) + 10(\underbrace{10 + I_1}_{I_2} - I_4) = 0$

$$-100 + 10I_1 + 100 + 10I_1 - 10I_3 + 100 + 10I_1 - 10I_4 = 0$$

$$(30)I_1 + (-10)I_3 + (-10)I_4 = -100 \quad -(1)$$

KVL equation for essential mesh 3: $10\{I_3 - (\underbrace{10 + I_1}_{I_2})\} + 10I_3 + 10(I_3 - I_4) = 0$

$$10I_3 - 100 - 10I_1 + 10I_3 + 10I_3 - 10I_4 = 0$$

$$(-10)I_1 + (30)I_3 + (-10)I_4 = 100 \quad -(2)$$

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KVL equation for essential mesh 4: $10 \times \{ I_4 - (\underbrace{10+I_1}_{I_2}) \} + 10(I_4 - I_3) + 24 I_4 = 0$

$$10 I_4 - 100 - 10 I_1 + 10 I_4 - 10 I_3 + 24 I_4 = 0$$

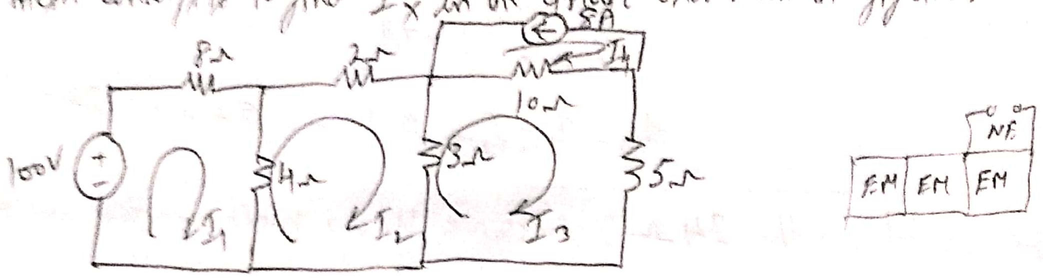
$$(-10) I_1 + (-10) I_3 + (44) I_4 = 100 \quad -③$$

Solving ①, ② & ③

$$x = I_1 = -1.029 A$$
$$y = I_3 = 3.970 A$$
$$z = I_4 = 2.941 A \text{ (current flowing through } 24\Omega)$$

$$I_2 = 10 + I_1 = 10 - 1.029 = 8.971 A$$

Prob: Use mesh analysis to find I_x in the circuit shown in the figure.



Sol:

KCL to non-essential mesh 4: $I_4 = -8$

In the remaining equations substitute $I_4 = -8$, the unknowns will be I_1, I_2, I_3.

KVL to essential mesh 1: $-100 + 8I_1 + 4(I_1 - I_2) = 0$

$$(12) I_1 + (-4) I_2 + (0) I_3 = 100 \quad -①$$

KVL to essential mesh 2: $4(I_2 - I_1) + 2I_2 + 3(I_2 - I_3) = 0$

$$(-4) I_1 + (9) I_2 + (-3) I_3 = 0 \quad -②$$

KVL to essential mesh 3: $3(I_3 - I_2) + 10\{ I_3 - (\underbrace{-8}_{I_4}) \} + 5 I_3 = 0$

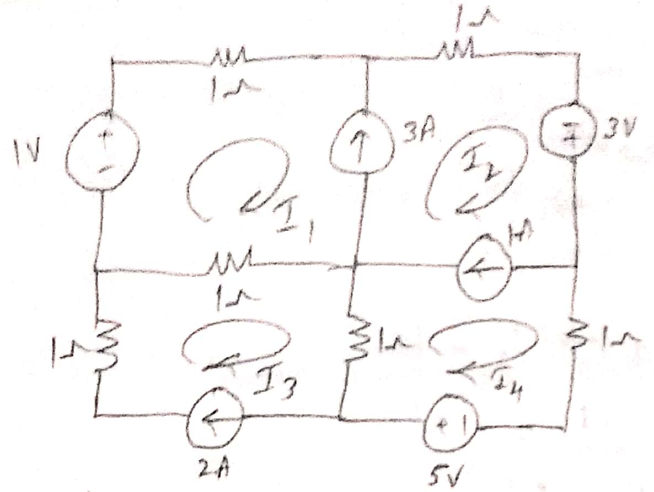
$$3I_3 - 3I_2 + 10 I_3 + 80 + 5 I_3 = 0$$

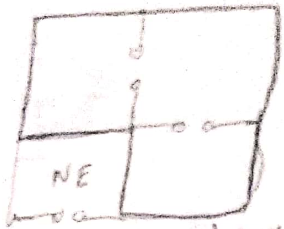
$$(0) I_1 + (-3) I_2 + (18) I_3 = -80 \quad -③$$

Solving ①, ② & ③

$$I_1 = 9.26 A \qquad I_4 = -8A$$
$$I_2 = 2.79 A$$
$$I_3 = -3.97 A$$

Prob: Find I_1, I_2, I_3, I_4 using mesh analysis





↓ KVL for supermesh 1, 2, 4

KCL equation for non-essential mesh 3: $I_3 = 2$

In the remaining equations substitute $I_3 = 2$, the unknowns will be I_1, I_2, I_4

KCL equation for supermesh 1, 2, 4 : $I_2 - I_1 = 3$

KCL equation for supermesh 1, 2, 4 : $I_2 - I = 1$

KVL equation for supermesh 1, 2, 4 : $-1 + 1I_1 + 1I_2 - 3 + 1I_4 - 5 + 1(I_4 - \underset{I_3}{2})$
$+ 1(I_1 - \underset{I_3}{2}) = 0$

-1
-3
-5
-2
-2
-13 (RHS)

$$(-1)I_1 + (1)I_2 + (0)I_4 = 3$$
$$(0)I_1 + (1)I_2 + (-1)I_4 = 1$$
$$(2)I_1 + (1)I_2 + (2)I_4 = 13$$

$x = I_1 = 1.2\,A$

$y = I_2 = 4.2\,A$ $\qquad I_2 = 2\,A$

$z = I_4 = 3.2\,A$

Inspection method

When all the meshes are essential meshes (i.e. Circuit with no current sources)
We can apply inspection method, based on $[R][I] = [V]$

For a 3 loop problem

$$\begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}$$

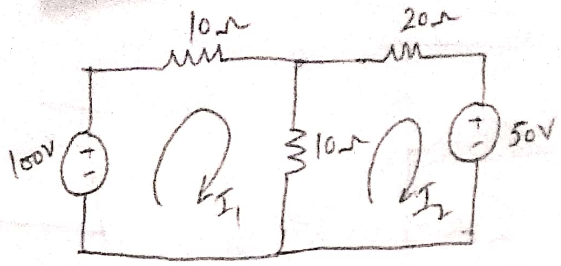
$V_1 \to$ first loop voltage
$V_2 \to$ Second loop voltage
$V_3 \to$ Third loop voltage

$R_{11} \to$ Total resistance in loop 1
$R_{22} \to$ Total resistance in loop 2
$R_{33} \to$ Total resistance in loop 3

$R_{12} = R_{21} = -$(resistance common to loop 1 & 2)
$R_{13} = R_{31} = -$(" 1 & 3)
$R_{23} = R_{32} = -$(" 2 & 3)

Prob: Solve for mesh currents for the circuit shown in figure.

Sol:



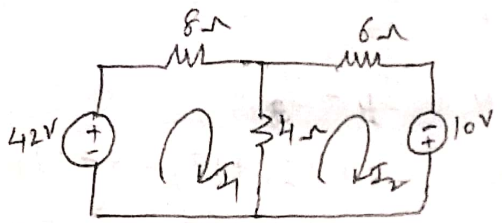
By inspection, $(20)I_1 + (-10)I_2 = 100$

$(-10)I_1 + (30)I_2 = -50$

$I_1 = 5A$

$I_2 = 0A$

Prob: Solve for mesh currents for the circuit shown in figure



Sol:

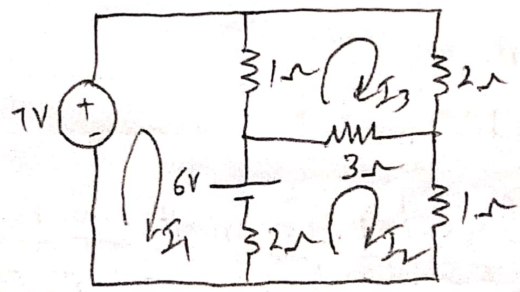
By inspection, $(12)I_1 + (-4)I_2 = 42$

$(-4)I_1 + (10)I_2 = 10$

$I_1 = 4.42A$

$I_2 = 2.76A$

Prob: For the circuit shown in figure, find the mesh currents.



Sol: By inspection, $(3)I_1 + (-2)I_2 + (-1)I_3 = 7-6$

$(-2)I_1 + (6)I_2 + (-3)I_3 = 6$

$(-1)I_1 + (-3)I_2 + (6)I_3 = 0$

$I_1 = 3A, \; I_2 = 3A, \; I_3 = 2A$

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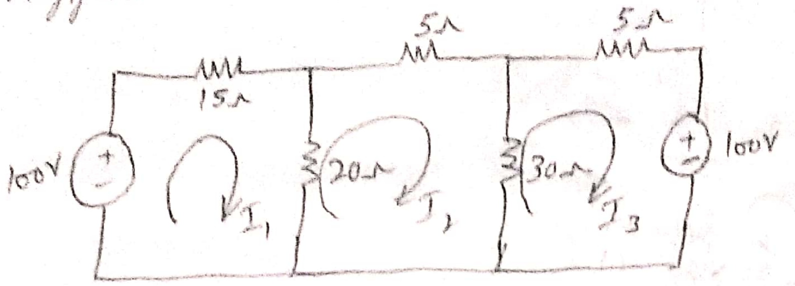
Prob: Find the current in 30 Ω resistance using mesh analysis for the circuit shown in the figure.



By inspection,

$$(35)I_1 + (-20)I_2 + (-0)I_3 = 100$$

$$(-20)I_1 + (55)I_2 + (-30)I_3 = 0$$

$$(-0)I_1 + (-30)I_2 + (35)I_3 = -100$$

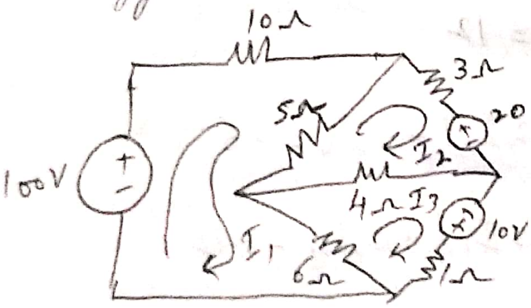
$$I_1 = 1.942\ A$$

$$I_2 = -1.6\ A$$

$$I_3 = -4.22\ A$$

$$\underset{(\downarrow)}{I_{30\Omega}} = I_2 - I_3 = (-1.6) - (-4.22) = 2.62\ A$$

Prob: Find the current in 4 Ω resistance using mesh analysis for the circuit shown in the figure.



By inspection,

$$(21)I_1 + (-5)I_2 + (-6)I_3 = 100$$

$$(-5)I_1 + (12)I_2 + (-4)I_3 = -20$$

$$(-6)I_1 + (-4)I_2 + (11)I_3 = 10$$

$$I_1 = 7.34\ A$$

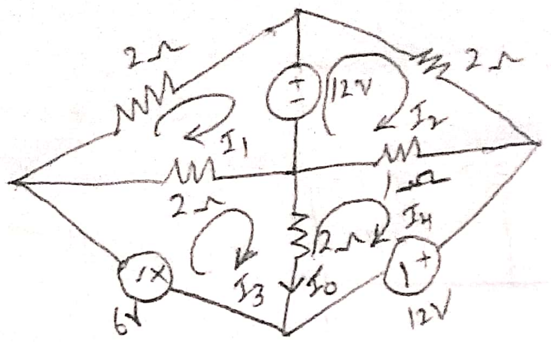
$$I_2 = 3.45\ A$$

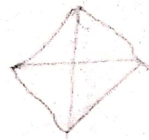
$$I_3 = 6.17\ A$$

$$\underset{(\leftarrow)}{I_{4\Omega}} = I_2 - I_3 = 3.45 - 6.17 = -2.72\ A$$

$$I_{4\Omega}(\rightarrow) = I_3 - I_2 = 2.72\ A$$

Prob: In the circuit shown in figure, find I_0 using mesh analysis.





Sol: By inspection,

$$(4)I_1 + (-0)I_2 + (-2)I_3 + (-0)I_4 = -12 \quad -①$$
$$(-0)I_1 + (3)I_2 + (-0)I_3 + (-1)I_4 = +12 \quad -②$$
$$(-2)I_1 + (-0)I_2 + (4)I_3 + (-2)I_4 = -6 \quad -③$$
$$(-0)I_1 + (-1)I_2 + (-2)I_3 + (3)I_4 = -12 \quad -④$$

From equation ①, $-2I_3 = -12 - 4I_1$

$$I_3 = 6 + 2I_1$$

Substitute for I_3 in eq ②, ③ & ④

Eq ② $\Rightarrow$ $3I_2 - 1I_4 = 12$

Eq ③ $\Rightarrow$ $-2I_1 + 4(6 + 2I_1) - 2I_4 = -6$

eq ④ $\Rightarrow$ $-1I_2 - 2(6 + 2I_1) + 3I_4 = -12$

Sorting the above equations,

$$(0)I_1 + (3)I_2 + (-1)I_4 = 12$$
$$(6)I_1 + (0)I_2 + (-2)I_4 = -30$$
$$(-4)I_1 + (-1)I_2 + (3)I_4 = 0$$

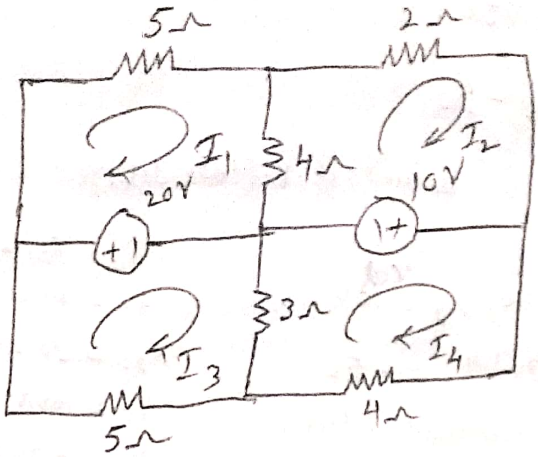
$x = I_1 = -9A$

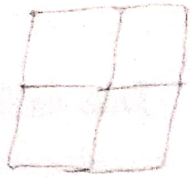
$y = I_2 = 0A$ $\qquad I_3 = 6 + 2I_1 = -12A$

$z = I_4 = -12A$

$I_0 = I_3 - I_4 = -12 - (-12) = 0A$

Prob: obtain the four mesh currents for the circuit shown in figure.





By inspection,

$$(9)I_1 + (-4)I_2 + (0)I_3 + (0)I_4 = 20 \quad -①$$
$$(-4)I_1 + (6)I_2 + (0)I_3 + (0)I_4 = -10 \quad -②$$
$$(0)I_1 + (0)I_2 + (8)I_3 + (-3)I_4 = -20 \quad -③$$
$$(0)I_1 + (0)I_2 + (-3)I_3 + (7)I_4 = 10 \quad -④$$

From equation ①, $4I_2 = 9I_1 - 20$

$$I_2 = 2.25 I_1 - 5$$

Substitute for I_2 in eq ②, ③ & ④

Eq ② ⇒ $-4I_1 + 6(2.25I_1 - 5) = -10$

Eq ③ ⇒ $8I_3 - 3I_4 = -20$

Eq ④ ⇒ $-3I_3 + 7I_4 = 10$

Sorting the above equations,

$$(9.5)I_1 + (0)I_3 + (0)I_4 = 20$$
$$(0)I_1 + (8)I_3 + (-3)I_4 = -20$$
$$(0)I_1 + (-3)I_3 + (7)I_4 = 10$$

$x = I_1 = 2.10A$

$y = I_3 = -2.34A$

$z = I_4 = 0.42A$

$I_2 = 2.25 I_1 - 5$

$= 2.25 \times 2.10 - 5$

$= -0.275A$

Prob: The mesh equations of a particular circuit are:

$$20I_1 - 10I_2 = 150$$

$$-10I_1 + 30I_2 = -50$$

Draw the circuit and insert the values of voltage sources and resistances.

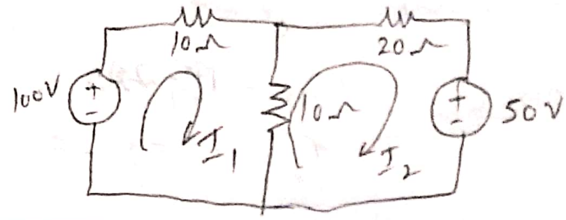
Sol:

$$(20)I_1 + (-10)I_2 = 100$$

$$(-10)I_1 + (30)I_2 = -50$$

$R_{12} = R_{21} = 10\,\Omega$ (Multiply the given value by -)
$R_{11} = 20\,\Omega$
$R_{22} = 30\,\Omega$
$V_1 = 100\,V$
$V_2 = -50\,V$

Procedure:
1) First write the common elements between meshes, $R_{12} = 10\,\Omega$
2) The remaining resistance in the first mesh is $R_{11} - R_{12} = 20 - 10 = 10\,\Omega$
   The remaining resistance in the second mesh is $R_{22} - R_{12} = 30 - 10 = 20\,\Omega$
3) Write the voltage source in the mesh 1, $V_1 = 100\,V$ (Taking care of polarity)
   Write the voltage source in the mesh 2, $V_2 = -50\,V$ (Taking care of polarity)
4) Write clockwise mesh currents.



Prob: The mesh equations of a particular circuit are;

$$11I_1 - 5I_2 = 50$$

$$-5I_1 + 27I_2 - 4I_3 = 0$$

$$-4I_2 + 8I_3 = 0$$

Draw the circuit.

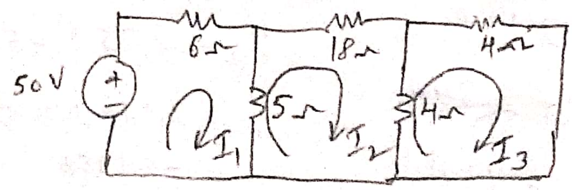
Sol:

$$(11)I_1 + (-5)I_2 + (0)I_3 = 50$$

$$(-5)I_1 + (27)I_2 + (-4)I_3 = 0$$

$$(0)I_1 + (-4)I_2 + (8)I_3 = 0$$

$R_{12} = R_{21} = 5\,\Omega$
$R_{13} = R_{31} = 0\,\Omega$
$R_{23} = R_{32} = 4\,\Omega$
$R_{11} = 11\,\Omega$, $R_{22} = 27$
$R_{33} = 8\,\Omega$
$V_1 = 50\,V$, $V_2 = 0\,V$
$V_3 = 0\,V$



When $R_{13} = R_{31} = 0\,\Omega$, the circuit can be written as above, there is no common element between mesh 1 & 3.

Prob: The mesh equations of a particular Circuit are:

$$21I_1 - 5I_2 - 6I_3 = 100$$

$$-5I_1 + 12I_2 - 4I_3 = -20$$

$$-6I_1 - 4I_2 + 11I_3 = 10$$

Draw the circuit.

Sol:

$$(21)I_1 + (-5)I_2 + (-6)I_3 = 100$$

$$(-5)I_1 + (12)I_2 + (-4)I_3 = -20$$

$$(-6)I_1 + (-4)I_2 + (11)I_3 = 10$$

$R_{12} = R_{21} = 5\Omega$

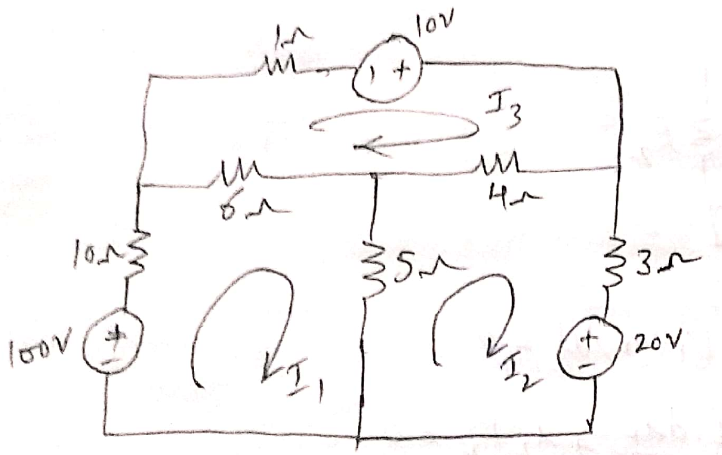
$R_{13} = R_{31} = 6\Omega$

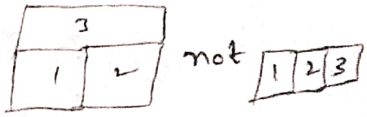
$R_{23} = R_{32} = 4\Omega$

$R_{11} = 21\Omega$ $V_1 = 100\,V$

$R_{22} = 12\Omega$ $V_2 = -20\,V$

$R_{33} = 11\Omega$ & $V_3 = 10\,V$





As $R_{13} = R_{31}$ is not zero

Loop Analysis (Mesh Analysis) (AC Excitation)

Note:

Time domain equations: $v = iR$ $\quad v(t) = V_m \cos/\sin(\omega t \pm \theta)$

R unit is Ω $\quad v = L \cdot \frac{di}{dt}$

L unit is H (henry) $\quad i(t) = I_m \cos/\sin(\omega t \pm \phi)$

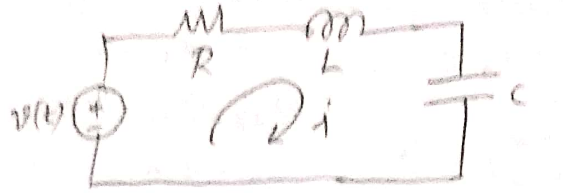
C unit is F (farad) $\quad v = \frac{1}{C}\int i\,dt$

v & i should be in lowercase

(Voltage source & current source expressions)

Prob: Write down the KVL equations in time domain for the following circuits

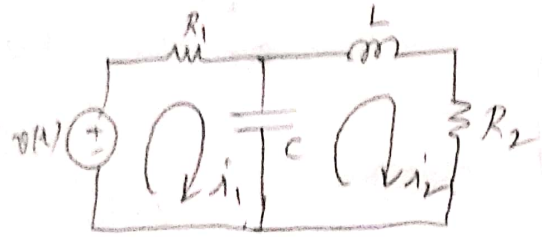
a)



$$-v(t) + iR + L\frac{di}{dt} + \frac{1}{C}\int i\,dt = 0$$

$$iR + L.\frac{di}{dt} + \frac{1}{C}\int i\,dt = v(t)$$

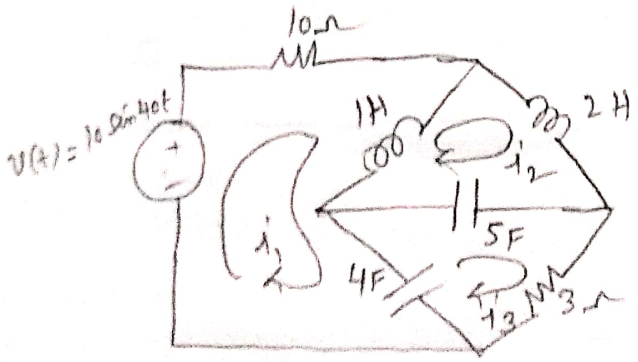
b)



KVL to loop 1: $-v(t) + i_1 R_1 + \frac{1}{C}\int (i_1 - i_2)\,dt = 0$

KVL to loop 2: $\frac{1}{C}\int (i_2 - i_1)\,dt + L.\frac{di_2}{dt} + i_2 R_2 = 0$

c)

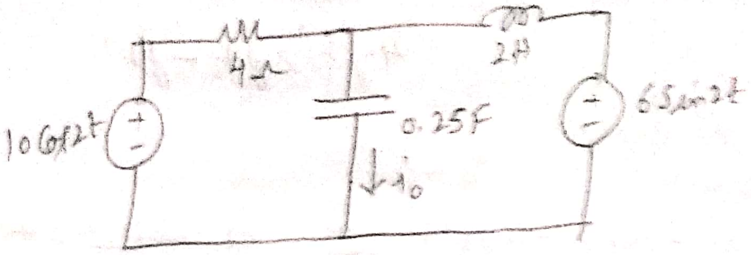


KVL to loop 1: $-10\sin 40t + 10 i_1 + 1.\frac{d}{dt}(i_1 - i_2) + \frac{1}{4}\int (i_1 - i_3)\,dt = 0$

KVL to loop 2: $1.\frac{d}{dt}(i_2 - i_1) + 2.\frac{d}{dt}(i_2 - i_1) + \frac{1}{5}\int (i_2 - i_3)\,dt = 0$

KVL to loop 3: $\frac{1}{4}\int (i_3 - i_1)\,dt + \frac{1}{5}\int (i_3 - i_2)\,dt + 3 i_3 = 0$

Prob: Solve for i_o using mesh analysis for the circuit shown in figure



Sol: The given circuit is in time domain. It is converted to frequency domain ckt.
Both the sources should be in 'Cos' or 'Sin'. From Sin to 'Cos' subtract 90° from angle.

$Z \to$ impedance
$R \to$ Resistance
$X_L \to$ inductive reactance
$X_C \to$ capacitive reactance

$Z = R + jX_L - jX_C$

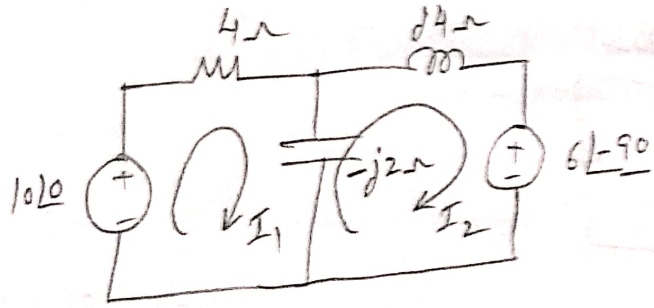
$X_L = \omega L = 2\pi f L$

$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$

Time domain	Frequency domain
$10 \cos 2t$	$10\angle 0$
$6 \sin 2t$	$6\angle 0 \Rightarrow 6\angle 0-90 = 6\angle -90$
4Ω	4Ω
$2H$	$+jX_L \Rightarrow +j\omega L \Rightarrow +j \times 2 \times 2 = +j4\Omega$
$0.25F$	$-jX_C \Rightarrow -j\frac{1}{\omega C} \Rightarrow -j\frac{1}{2 \times 0.25} = -j2\Omega$

Now both the sources are in Cos

coefficient of t in the voltage notation



By inspection method, $[Z][I] = [V]$

$$\underbrace{(4-j2)}_{Z_{11}} I_1 + \underbrace{(+j2)}_{Z_{12}} I_2 = 10\angle 0$$

$$\underbrace{(+j2)}_{Z_{21}} I_1 + \underbrace{(j2)}_{Z_{22}} I_2 = -6\angle -90$$

off-diagonal we have to multiply the element by '$-$'. so $-(-j2) = +j2$

By Cramer's rule

Keep calculator in complex & degree mode

$$I_1 = \frac{\begin{vmatrix} 10\angle 0 & j2 \\ -6\angle -90 & j2 \end{vmatrix}}{\begin{vmatrix} (4-j2) & j2 \\ j2 & j2 \end{vmatrix}} = \frac{10 \times j2 + 6\angle -90 \times j2}{(4-j2) \times j2 - j2 \times j2} = \frac{(12+j20)}{(8+j8)}$$

$I_1 = 2 + j0.5 = 2.06\angle 14.03$ A (In frequency domain)

$i_1 = 2.06 \cos(2t + 14.03)$ A (In time domain)

$$I_2 = \frac{\begin{vmatrix} (4-j2) & 10\angle 0 \\ j2 & -6\angle -90 \end{vmatrix}}{\begin{vmatrix} (4-j2) & j2 \\ j2 & j2 \end{vmatrix}} = \frac{-(4-j2)\times 6\angle -90 - j2\times 10}{(8+j8)}$$

(No change in Dr)

Note: In the numerator shift the '−' sign to $-(4-j2)\times 6\angle -90$ or $-6\angle -90 = +6\angle -90 \pm 180 = 6\angle 90$

If we write $(4-j2)\times \underset{\text{negative magnitude}}{\underbrace{-6\angle -90}}$ calculator will show Math Error

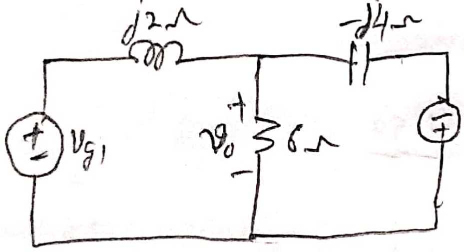
$$I_2 = \frac{(12+j4)}{(8+j8)} = 1-j0.5 = 1.118\angle -26.56 \text{ A}$$ (Frequency domain)

$$i_2 = 1.118 \cos(2t - 26.56^\circ)$$ (Time domain)

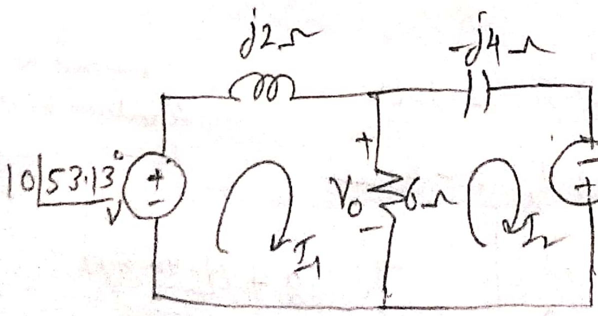
$$I_0 = I_1 - I_2 = (2+j0.5)-(1-j0.5) = (1+j1) = 1.414\angle 45^\circ \text{ A}$$ (Frequency domain)

$$i_0 = 1.414 \cos(2t+45^\circ)$$ (Time domain)

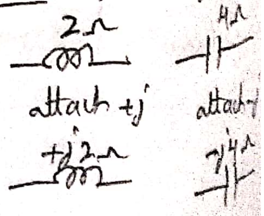
Prob. Solve the following example by mesh current method given $v_{g_1} = 10\cos(5000t+53.13)$, $v_{g_2} = 8\sin 5000t$. Find v_0.



Sol:



Note: If it is given as



BY inspection method

$$(6+j2)I_1 + (-6)I_2 = 10\angle 53.13$$

$$(-6)I_1 + (6-j4)I_2 = 8\angle -90$$

By Cramer's rule, $I_1 = \dfrac{\begin{vmatrix} 10\angle 53.13 & -6 \\ 8\angle -90 & (6-j4) \end{vmatrix}}{\begin{vmatrix} (6+j2) & -6 \\ -6 & (6-j4) \end{vmatrix}} = \dfrac{10\angle 53.13 \times (6-j4) + 6\times 8\angle -90}{(6+j2)(6-j4) - 6\times 6}$

$$I_1 = \frac{(68-j24)}{(8-j12)} = 4+j3 = 5\angle 36.86\ A$$

$i_1 = 5\cos(5000t + 36.86^\circ)$ (Time domain)

$$I_2 = \frac{\begin{vmatrix} (6+j2) & 10\angle 53.13 \\ -6 & 8\angle -90 \end{vmatrix}}{(8-j12)} = \frac{(6+j2)\times 8\angle -90 + 6\times 10\angle 53.13}{(8-j12)}$$

$$I_2 = \frac{52}{(8-j12)} = 2+j3 = 3.6\angle 56.3^\circ$$

$$i_2 = 3.6\cos(5000t + 56.3^\circ)$$

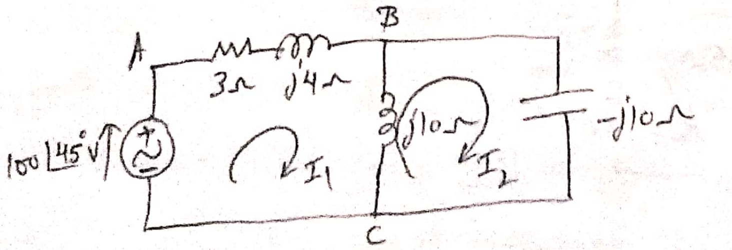
$$V_o = 6\times(I_1 - I_2)$$

$$V_o = 6\times(4+j3-2-j3) = 12\ V$$

$$v_o = 12\cos(5000t)$$

Prob. For the network shown in figure find V_{AB} & V_{BC} using mesh analysis

Sol:



Note:

If voltage source is shown with an arrow mark, mark + polarity for the tip of arrow mark

By inspection, $(3+j14)I_1 + (-j10)I_2 = 100\angle 45$

$(-j10)I_1 + (0)I_2 = 0$

By Cramer's rule,
$$I_1 = \frac{\begin{vmatrix} 100\angle 45 & -j10 \\ 0 & 0 \end{vmatrix}}{\begin{vmatrix} (3+j14) & -j10 \\ -j10 & 0 \end{vmatrix}} = \frac{0}{\Delta} = 0A$$

$$I_2 = \frac{\begin{vmatrix} (3+j14) & 100\angle 45 \\ -j10 & 0 \end{vmatrix}}{\begin{vmatrix} (3+j14) & -j10 \\ -j10 & 0 \end{vmatrix}}$$

$$I_2 = \frac{j10 \times 100\angle 45}{-j10 \times j10} = \frac{(-707.10 + j707.10)}{100}$$

$$I_2 = -7.071 + j7.071 = 10\angle 135\ A$$

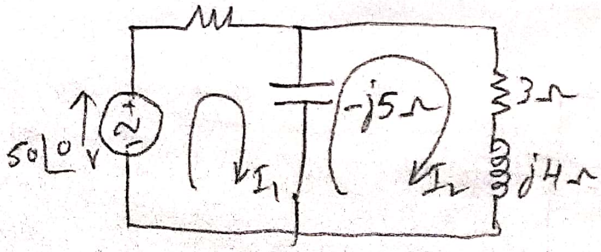
$V_{AB} = (3+j4) \times I_1 = (3+j4) \times 0 = 0$

$V_{BC} = j10 \times (I_1 - I_2) = j10 \times -10\angle 135 = -j10 \times 10\angle 135 = 70.71 + j70.71$

$V_{BC} = 100\angle 45$

Prob. Find the power output of the voltage source in the circuit shown in the figure and also determine the power in the circuit resistors using mesh analysis

Sol:



By inspection method

$$(10-j5)I_1 + (+j5)I_2 = 50\angle 0$$
$$(+j5)I_1 + (3-j1)I_2 = 0$$

By Cramer's rule, $I_1 = \dfrac{\begin{vmatrix} 50\angle 0 & j5 \\ 0 & (3-j1) \end{vmatrix}}{\begin{vmatrix} (10-j5) & j5 \\ j5 & (3-j1) \end{vmatrix}} = \dfrac{50 \times (3-j1)}{(10-j5)\times(3-j1) - j5 \times j5}$

$$I_1 = \frac{(150-j50)}{(50-j25)} = 2.8+j0.4 = \underset{|I_1|}{2.828}\angle 8.13$$

$$I_2 = \frac{\begin{vmatrix} (10-j5) & 50\angle 0 \\ j5 & 0 \end{vmatrix}}{(50-j25)} = \frac{-j5\times 50}{(50-j25)} = \frac{-j250}{(50-j25)}$$

$$I_2 = 2-j4 = \overset{|I_2|}{4.47}\angle -63.43$$

Take only Magnitude of voltage & current

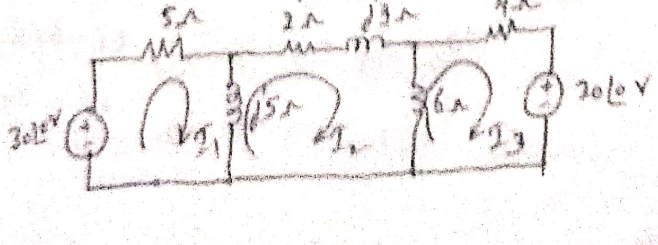
Power delivered by voltage source, $P_{50\angle 0 V} = |V|\,|I_1| = 50 \times 2.828 = 141.4W$

$$P_{10\Omega} = |I_1|^2 \times 10 = 2.828^2 \times 10 = 80W$$
$$P_{3\Omega} = |I_2|^2 \times 3 = 4.47^2 \times 3 = 60W$$

$P_{j4\Omega} = 0W$
$P_{-j5\Omega} = 0W$

pure inductor & pure capacitor does not consume any power.

Prob: The network shown in the figure contains two voltage sources. Find the current in the $(2+j3)\Omega$ impedance using loop analysis



By inspection method,

$$(5+j5)I_1 + (-j5)I_2 + (0)I_3 = 30\angle 0$$
$$(-j5)I_1 + (8+j8)I_2 + (-6)I_3 = 0$$
$$(0)I_1 + (-6)I_2 + (10)I_3 = -20\angle 0$$

Current through $(2+j3)\Omega$ is I_2. We need to find only I_2 not I_1 & I_3.

By Cramer's rule,

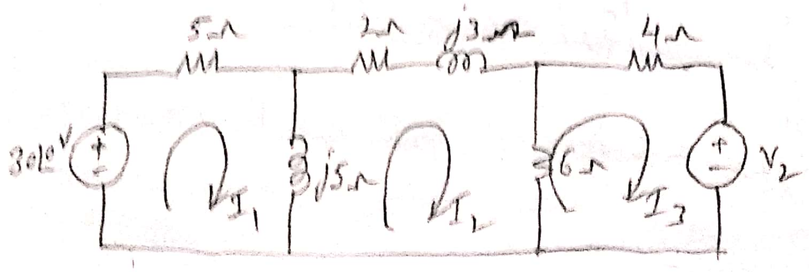
$$I_2 = \frac{\begin{vmatrix} (5+j5) & 30\angle 0 & 0 \\ -j5 & 0 & -6 \\ 0 & -20\angle 0 & 10 \end{vmatrix}}{\begin{vmatrix} (5+j5) & -j5 & 0 \\ -j5 & (8+j8) & -6 \\ 0 & -6 & 10 \end{vmatrix}} = \frac{(5+j5)\times -6\times 20 - 30\times -j5\times 10}{(5+j5)\times\{(8+j8)\times 10 - 6\times 6\} + j5\times 10\times -j5}$$

$$I_2 = \frac{(-600+j900)}{(5+j5)\times(44+j80)+250} = \frac{(-600+j900)}{(70+j620)}$$

$$I_2 = 1.325 + j1.117 = 1.73\angle 40.13 \text{ A}$$

Prob: In the network shown in figure determine V_2 such that the current in the impedance $(2+j3)\Omega$ is zero. Use mesh analysis.

Sol:



By inspection method

$$(5+j5)I_1 + (-j5)I_2 + (0)I_3 = 30\angle 0$$
$$(-j5)I_1 + (8+j8)I_2 + (-6)I_3 = 0$$
$$(0)I_1 + (-6)I_2 + (10)I_3 = -V_2$$

Given $I_{(2+j3)\Omega} = 0$ which means $I_2 = 0$

By Cramer's rule,

$$I_2 = \frac{\begin{vmatrix} (5+j5) & 30 & 0 \\ -j5 & 0 & -6 \\ 0 & -V_2 & 10 \end{vmatrix}}{\begin{vmatrix} (5+j5) & -j5 & 0 \\ -j5 & (8+j8) & -6 \\ 0 & -6 & 10 \end{vmatrix}} = 0$$

$$\begin{vmatrix} (5+j5) & 30 & 0 \\ -j5 & 0 & -6 \\ 0 & -V_2 & 10 \end{vmatrix} = 0$$

$$(5+j5) \times -6V_2 - 30 \times -j5 \times 10 = 0$$

$$(5+j5) \times -6V_2 + j1500 = 0$$

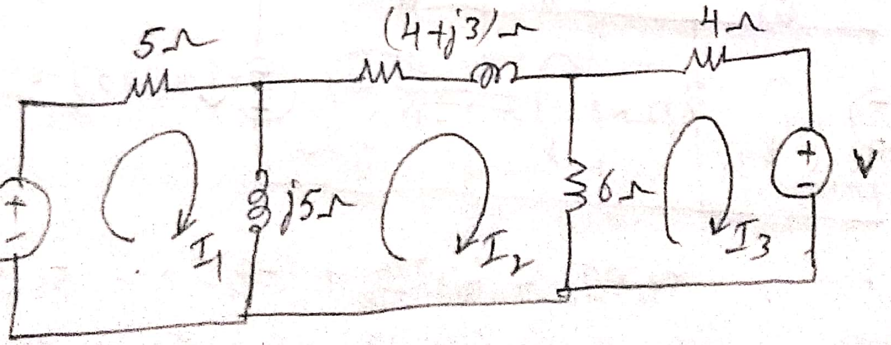
$$(5+j5) \times -6V_2 = -j1500$$

$$V_2 = \frac{-j1500}{-6 \times (5+j5)} = \frac{+j1500}{(+30+j30)}$$

$$V_2 = 25+j25 = 35.35\angle 45$$

Prob. Use loop analysis to determine voltage 'V' such that current through $(4+j3)\Omega$ is zero.

Sol:



By inspection method

$$(5+j5)I_1 + (-j5)I_2 + (-0)I_3 = 50\angle 0$$
$$(-j5)I_1 + (10+j8)I_2 + (-6)I_3 = 0$$
$$(-0)I_1 + (-6)I_2 + (10)I_3 = -V$$

Given $I_{4+j3\Omega} = 0$ which means $I_2 = 0$

$$\frac{\begin{vmatrix} (5+j5) & 50\angle 0 & 0 \\ -j5 & 0 & -6 \\ 0 & -V & 10 \end{vmatrix}}{D_T} = 0$$

$$(5+j5)\times -6V - 50\times -j5\times 10 = 0$$

$$(5+j5)\times -6V + j2500 = 0$$

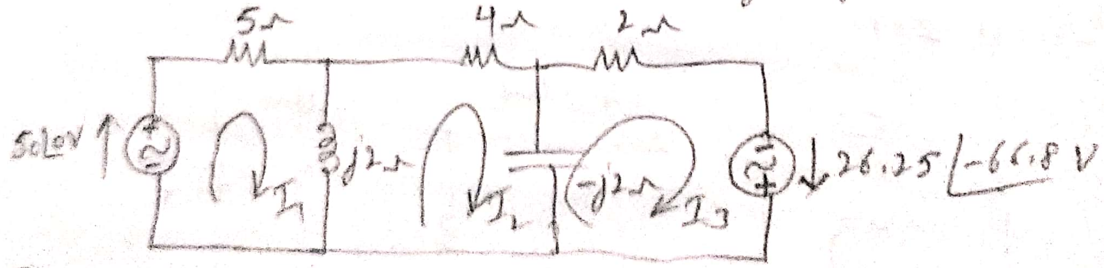
$$(5+j5)\times -6V = -j2500$$

$$V = \frac{-j2500}{(5+j5)\times -6} = \frac{-j2500}{(-30-j30)}$$

$$V = 41.66 + j41.66$$

$$V = 58.92\angle 45$$

Prob. Find current through 4Ω resistor by using loop current method



By inspection

$$(5+j2)I_1 + (-j2)I_2 + (-0)I_3 = 50\angle 0$$
$$(-j2)I_1 + (4)I_2 + (+j2)I_3 = 0$$
$$(-0)I_1 + (+j2)I_2 + (2-j2)I_3 = 26.25\angle -66.8$$

Current through 4Ω is I_2.

$$I_2 = \frac{\begin{vmatrix} (5+j2) & 50 & 0 \\ -j2 & 0 & j2 \\ 0 & 26.25\angle -66.8 & (2-j2) \end{vmatrix}}{D_r} = \frac{(5+j2)\times -j2 \times 26.25\angle -66.8 - 50\times -j2\times(2-j2)}{D_r}$$

$$I_2 = \frac{\cancel{(-200-j200)}+\cancel{(200+j200)}}{D_r} = 0\ A$$

Prob: The mesh equation of a given network are;

$$(8-j2)I_1 - 3I_2 = 10\angle 30$$
$$-3I_1 + (8+j5)I_2 - 5I_3 = 0$$
$$-5I_2 + (7-j2)I_3 = 0$$

Draw the network satisfying the above equations.

Sol:

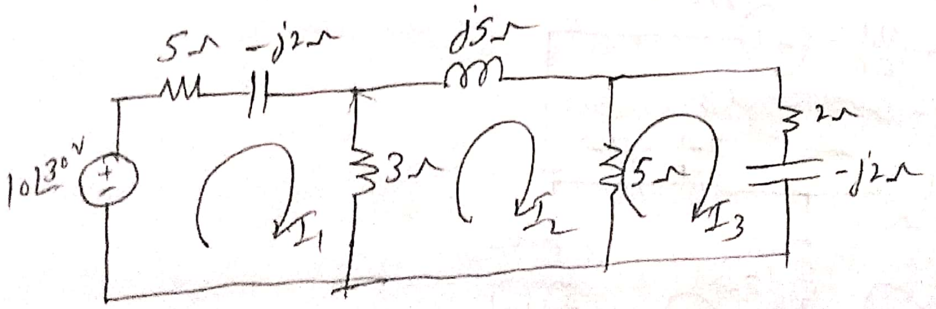
$$(8-j2)I_1 + (-3)I_2 + (0)I_3 = 10\angle 30$$
$$(-3)I_1 + (8+j5)I_2 + (-5)I_3 = 0$$
$$(0)I_1 + (-5)I_2 + (7-j2)I_3 = 0$$

Multiply by '$-$' for off diagonal elements,

$Z_{12} = Z_{21} = 3\Omega$ $\quad V_1 = 10\angle 30$ $\quad Z_{11} = (8-j2)$

$Z_{13} = Z_{23} = 0$ $\quad 0$ $\quad Z_{22} = (8+j5)$

$Z_{23} = Z_{32} = 5$ $\quad 0$ $\quad Z_{33} = (7-j2)$



Prob: Draw a network for the following mesh equations in matrix form

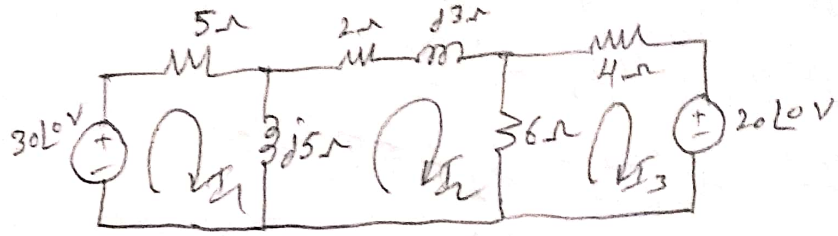
$$\begin{bmatrix} (5+j5) & -j5 & 0 \\ -j5 & (8+j8) & -6 \\ 0 & -6 & 10 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 30\angle 0 \\ 0 \\ -20\angle 0 \end{bmatrix}$$

Sol:

$Z_{12} = Z_{21} = +j5\,\Omega$ $\quad Z_{11} = 5+j5$ $\quad V_1 = 30\angle 0$

$Z_{13} = Z_{31} = 0\,\Omega$ $\quad Z_{22} = 8+j8$ $\quad V_2 = 0$

$Z_{23} = Z_{32} = 6\,\Omega$ $\quad Z_{33} = 10$ $\quad V_3 = -20\angle 0$



Prob: From the following representation of the matrix form of loop equations, draw the corresponding network

$$\begin{bmatrix} (7+j3) & -j5 & -5 \\ -j5 & (12+j3) & -2+j2 \\ -5 & -2+j2 & (17-j2) \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 10\angle 0 \\ -15\angle 0 \\ -25\angle 0 \end{bmatrix}$$

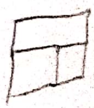
Sol:

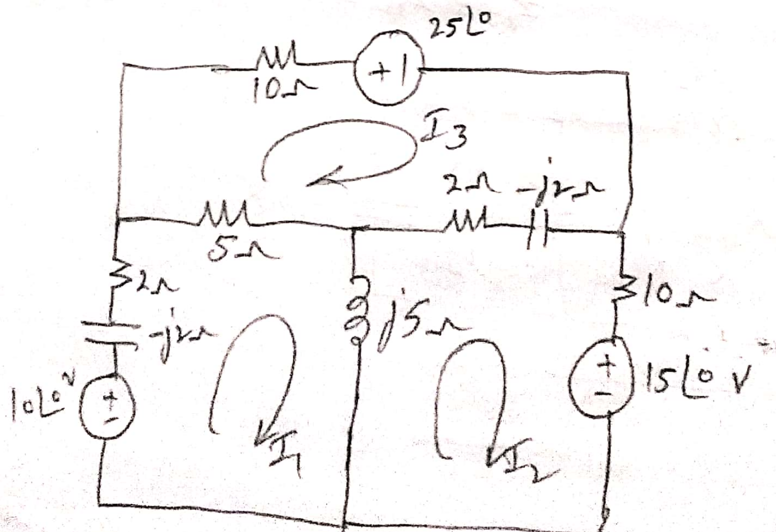
$Z_{12} = Z_{21} = j5$ $\quad Z_{11} = 7+j3$ $\quad V_1 = 10\angle 0$

$Z_{13} = Z_{31} = 5$ $\quad Z_{22} = 12+j3$ $\quad V_2 = -15\angle 0$

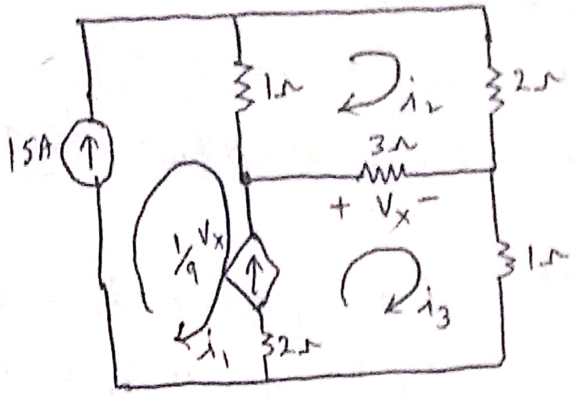
$Z_{23} = Z_{32} = 2-j2$ $\quad Z_{33} = (17-j2)$ $\quad V_3 = -25\angle 0$

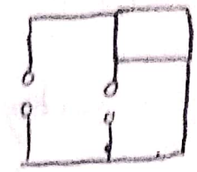
Since Z_{13} is not zero the n/w looks like





Find the three unknown currents in the circuit shown in figure using mesh Analysis. (8 Marks)





$$V_x = 3(i_3 - i_2)$$

KCL to non essential mesh 1 : $i_1 = +15$

KCL to non essential mesh 3 : $i_3 - i_1 = \frac{1}{9_3}\{\cancel{3}(i_3 - i_2)\} \Rightarrow 3(i_3 - i_1) = i_3 - i_2$

KVL to essential mesh 2 : $1(i_2 - i_1) + 2i_2 + 3(i_2 - i_3) = 0$

Sorting the above equations :

$$(1)i_1 + (0)i_2 + (0)i_3 = 15$$

$$(-3)i_1 + (1)i_2 + (2)i_3 = 0$$

$$(-1)i_1 + (6)i_2 + (-3)i_3 = 0$$

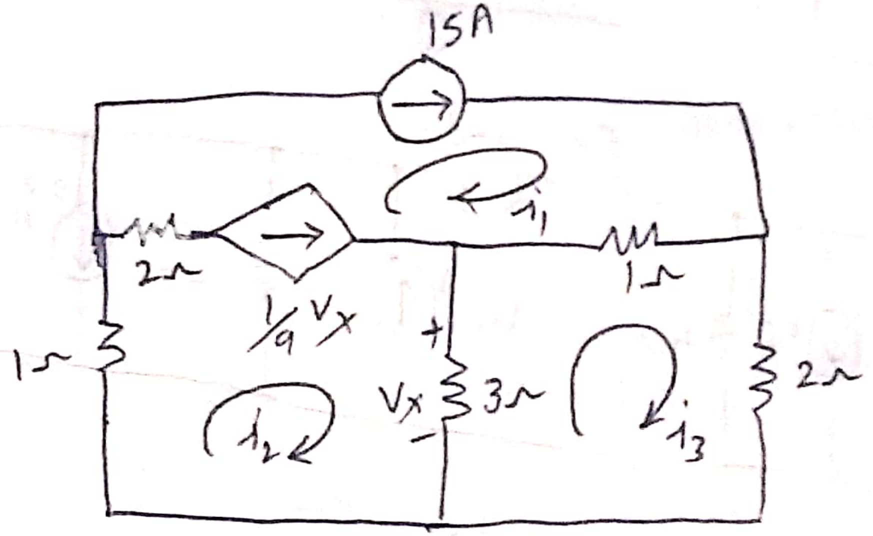
$$i_1 = 15\,A$$

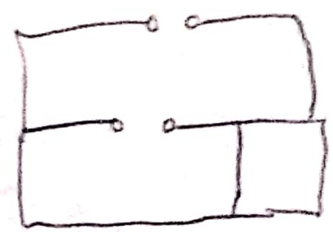
$$i_2 = 11\,A$$

$$i_3 = 17\,A$$

c. Find the loop currents i_1, i_2 and i_3 using mesh analysis for the network shown in figure. (6 Marks)

Sol:





$$V_x = 3(i_2 - i_3)$$

KCL to non essential mesh 1 : $i_1 = 15$

KCL to non essential mesh 2 : $i_2 - i_1 = \frac{1}{\cancel{9}_{3}}\{\underbrace{3(i_2 - i_3)}_{V_x}\} \Rightarrow 3(i_2 - i_1) = i_2 - i_3$

KVL to essential mesh 3 : $3(i_3 - i_2) + 1(i_3 - i_1) + 2i_3 = 0$

$$(1)i_1 + (0)i_2 + (0)i_3 = 15$$

$$(-3)i_1 + (2)i_2 + (1)i_3 = 0$$

$$(-1)i_1 + (-3)i_2 + (6)i_3 = 0$$

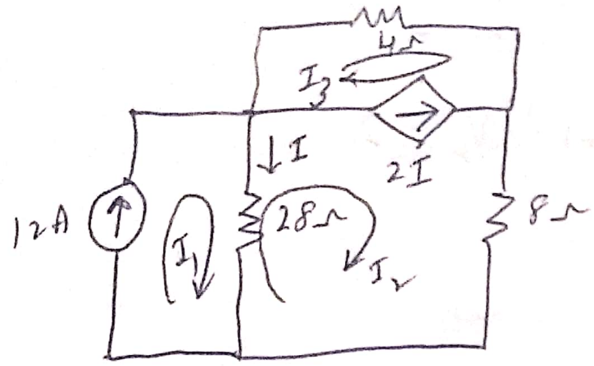
$$i_1 = 15A$$

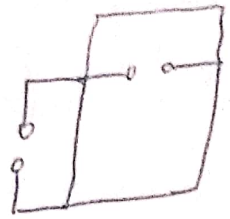
$$i_2 = 17A$$

$$i_3 = 11A$$

47

Find the current I using mesh Analysis for the Circuit shown in figure. (8 marks)





$$I = I_1 - I_2$$

KCL to nonessential mesh 1 : $I_1 = 12$

KCL to supermesh 2 & 3 : $I_2 - I_3 = 2\underbrace{(I_1 - I_2)}_{I}$

KVL to supermesh 2 & 3 : $28(I_2 - I_1) + 4I_3 + 8I_2 = 0$

$$(1)I_1 + (0)I_2 + (0)I_3 = 12$$

$$(-2)I_1 + (3)I_2 + (-1)I_3 = 0$$

$$(-28)I_1 + (36)I_2 + (4)I_3 = 0$$

$$I_1 = 12A$$

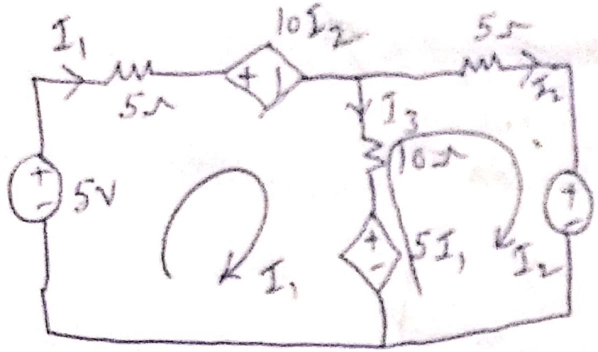
$$I_2 = 9A$$

$$I_3 = 3A$$

$$I = I_1 - I_2 = 12 - 9 = 3A$$

c. Use mesh analysis to determine the branch currents in the network indicated in the figure (6 marks)

Sol:



KVL to essential mesh 1 : $-5+5I_1+10I_2+10(I_1-I_2)+5I_1=0$

KVL to essential mesh 2 : $-5I_1+10(I_2-I_1)+5I_2+10=0$

$$(20)I_1+(0)I_2=5$$

$$(-15)I_1+(15)I_2=-10$$

$$I_1=0.25\,A$$

$$I_2=-0.416\,A$$

$$I_3=I_1-I_2=0.25+0.416$$

$$=0.666\,A$$

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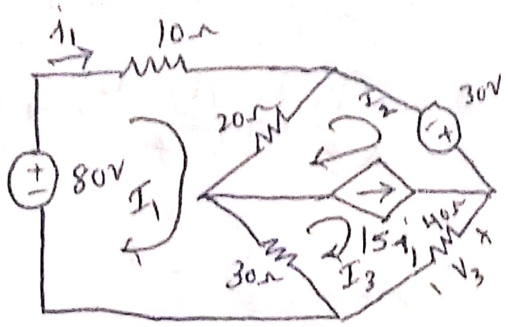
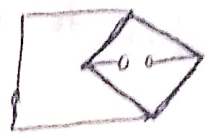
a. Determine Voltage V_3 for the Circuit shown in figure using mesh analysis method.
(8 Marks)





$i_1 = I_1$

KCL to supermesh 2 & 3: $I_3 - I_2 = 15 I_1$

KVL to supermesh 2 & 3: $30(I_3 - I_1) + 20(I_2 - I_1) - 30 + 40 I_3 = 0$

KVL to essential mesh 1: $-80 + 10 I_1 + 20(I_1 - I_2) + 30(I_1 - I_3) = 0$

$$(-15) I_1 + (-1) I_2 + (1) I_3 = 0$$

$$(-50) I_1 + (20) I_2 + (70) I_3 = 30$$

$$(60) I_1 + (-20) I_2 + (-30) I_3 = 80$$

$$I_1 = 0.5838 \text{ A}$$

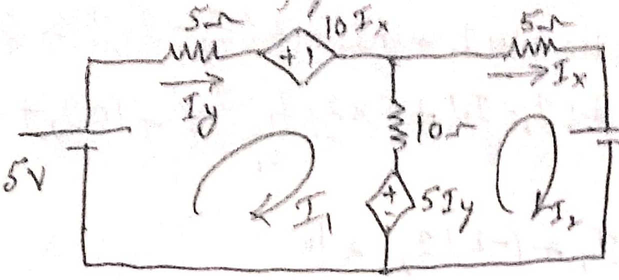
$$I_2 = -6.15 \text{ A}$$

$$I_3 = 2.60 \text{ A}$$

$$V_3 = +40 I_3 = 40 \times 2.6 = 104 \text{ V}$$

... V_3 for the Circuit

Q. Using mesh analysis find the current through 10 ohm resistor in the network shown in the figure.



Sol: Control variable, $I_x = +I_2$

$I_y = +I_1$

KVL to essential mesh 1: $-5 + 5I_1 + 10I_2 + 10(I_1 - I_2) + 5I_1 = 0$

KVL to essential mesh 2: $-5I_1 + 10(I_2 - I_1) + 5I_2 + 10 = 0$

$$(20)I_1 + (0)I_2 = 5$$

$$(-15)I_1 + (15)I_2 = -10$$

$$I_1 = 0.25A$$

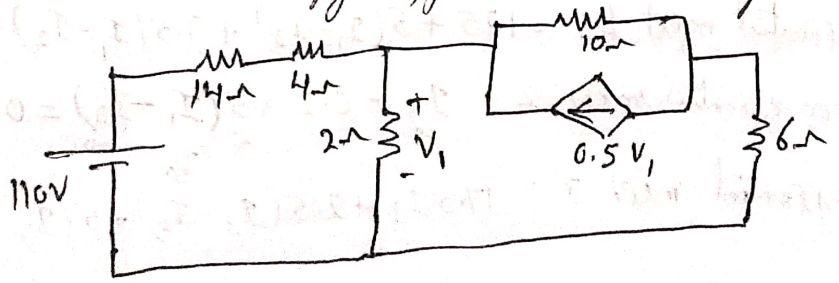
$$I_2 = -0.4166A$$

$$I_{10\Omega} = I_1 - I_2$$

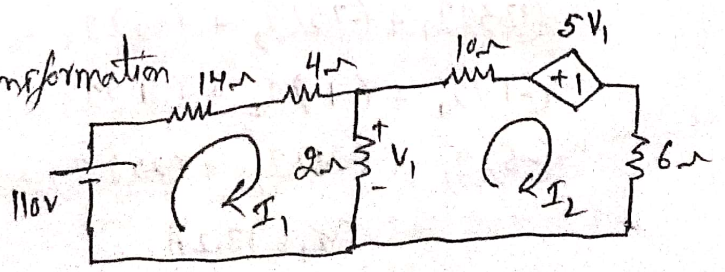
$$= 0.25 - (-0.4166)$$

$$= 0.666A$$

6. For the network shown in the figure, find the current through 4Ω and 6Ω resistors.



Sol: Using source transformation



Control Variable, $V_1 = 2(I_1 - I_2)$

entering '+' terminal

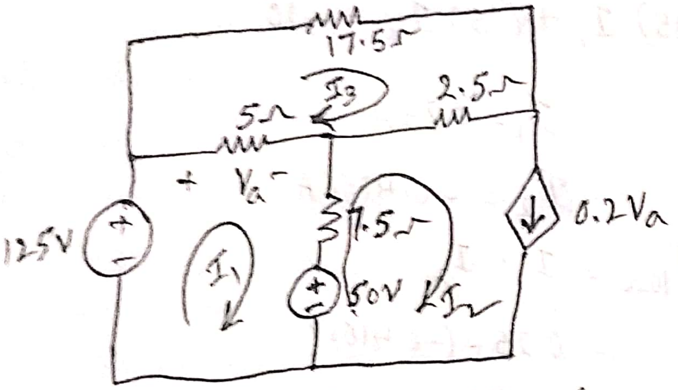
KVL to effential mesh 1 : $-110 + 14I_1 + 4I_1 + 2(I_1 - I_2) = 0$

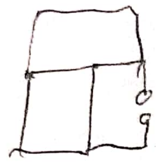
KVL to effential mesh 2 : $2(I_2 - I_1) + 5 \times \underbrace{2(I_1 - I_2)}_{V_1} + 10I_2 + 6I_2 = 0$

$$(20)\,I_1 + (-2)\,I_2 = 110$$
$$(-2)\,I_1 + (-2)\,I_2 = 0$$
$$I_1 = 5A$$
$$I_2 = -5A$$
$$I_{4\Omega} = I_1 = 5A$$
$$I_{6\Omega} = I_2 = -5A$$

Q. Find the mesh currents for the circuit shown in figure.

Sol:





Control Variable, $V_a = 5(I_1 - I_3)$

entering '+'

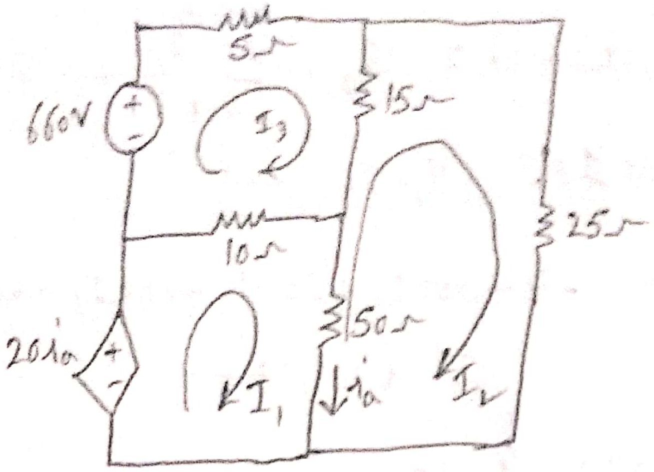
KVL to effential mesh 1 : $-125 + 5(I_1 - I_3) + 7.5(I_1 - I_2) + 50 = 0$

KCL to non-effential mesh 2 : $I_2 = 0.2 \times \underbrace{5(I_1 - I_3)}_{V_a}$

KVL to effential mesh 3 : $17.5I_3 + 2.5(I_3 - I_2) + 5(I_3 - I_1) = 0$

$$(12.5)\,I_1 + (-7.5)\,I_2 + (-5)\,I_3 = 75$$
$$(-1)\,I_1 + (1)\,I_2 + (1)\,I_3 = 0$$
$$(-5)\,I_1 + (-2.5)\,I_2 + (25)\,I_3 = 0$$
$$I_1 = 13.2A$$
$$I_2 = 9.6A$$
$$I_3 = 3.6A$$

Q. Find i_a in the circuit shown in figure using mesh analysis



Sol: Control variable, $i_a = I_1 - I_2$

KVL to essential mesh 1 : $-20\underbrace{(I_1 - I_2)}_{i_a} + 10(I_1 - I_3) + 50(I_1 - I_2) = 0$

KVL to essential mesh 2 : $50(I_2 - I_1) + 15(I_2 - I_3) + 25I_2 = 0$

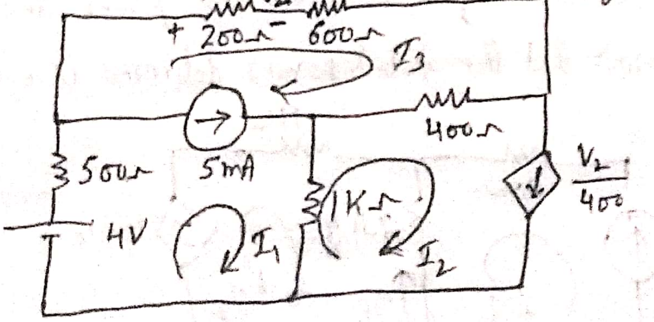
KVL to essential mesh 3 : $-660 + 5I_3 + 15(I_3 - I_2) + 10(I_3 - I_1) = 0$

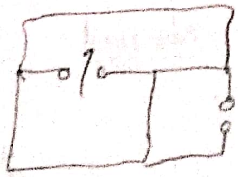
$$(40)I_1 + (-30)I_2 + (-10)I_3 = 0$$
$$(-50)I_1 + (90)I_2 + (-15)I_3 = 0$$
$$(-10)I_1 + (-15)I_2 + (30)I_3 = 660$$

$$I_1 = 27A$$
$$I_2 = 22A$$
$$I_3 = 42A$$

$$i_a = I_1 - I_2 = 27 - 22 = 5A$$

Q. Solve for mesh currents for the circuit shown in figure. Find the power supplied by 4V





Control Variable $V_2 = +200 I_3$

KCL to non-essential mesh 2 : $I_2 = \frac{200 I_3}{400} \Rightarrow 2I_2 = I_3$

KCL to supermesh 1 & 3 : $I_1 - I_3 = 5\times10^{-3}$

KVL to supermesh : $-4 + 500 I_1 + 200 I_3 + 600 I_3 + 400(I_3 - I_2) + 1K(I_1 - I_2) = 0$

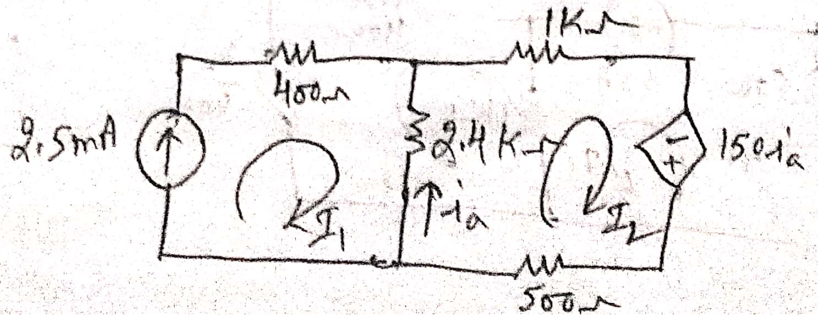
$$(0)I_1 + (2)I_2 + (-1)I_3 = 0$$
$$(1)I_1 + (0)I_2 + (-1)I_3 = 5\times10^{-3}$$
$$(1500)I_1 + (-1400) + (1200)I_3 = 4$$

$$I_1 = 3.25\text{ mA}$$
$$I_2 = -8.75\times10^{-4}\text{A} = -0.875\text{ mA}$$
$$I_3 = -1.75\text{ mA}$$

$$P_{absorbed(4V)} = \overset{\text{current entering '-ve' terminal}}{-}(V)(I_1)$$
$$= -(4)(3.25\times10^{-3})$$
$$= -0.013\text{ W}$$
$$= -13\text{ mW}$$

$$P_{delivered(4V)} = +13\text{ mW}$$

Q. Solve for mesh currents I_1 & I_2 and obtain the power delivered/absorbed by all the elements. Prove that the total power delivered is equal to total power absorbed.



Control variable $i_a = I_2 - I_1$

KCL for non-essential mesh 1, $I_1 = 2.5\times10^{-3}$

KVL for essential mesh 2 : $2.4K(I_2 - I_1) + 1KI_2 - 150\underbrace{(I_2 - I_1)}_{i_a} + 500I_2 = 0$

$$(1)I_1 + (0)I_2 = 2.5\times10^{-3}$$

$$(-2250)I_1 + (3750)I_2 = 0$$

$$I_1 = 2.5\,mA$$

$$i_a = I_2 - I_1 = -1\,mA$$

$$I_2 = 1.5\,mA$$

Power will always be absorbed by resistances.

$$P_{400\Omega} = I_1^2 \times 400 = (2.5\times10^{-3})^2 \times 400 = 2.5\times10^{-3}W$$

$$P_{500\Omega} = I_2^2 \times 500 = (1.5\times10^{-3})^2 \times 500 = 1.125\times10^{-3}W$$

$$P_{2.4K\Omega} = (I_2 - I_1)^2 \times 2.4\times10^3 = \{(1.5-2.5)\times10^{-3}\}^2 \times 2.4\times10^3 = 2.4\times10^{-3}W$$

or $(I_1 - I_2)^2$

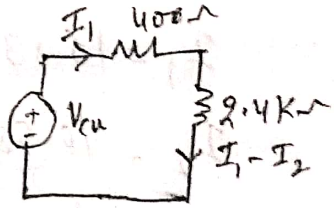
$$P_{1K\Omega} = I_2^2 \times 1\times10^3 = (1.5\times10^{-3})^2 \times 1\times10^3 = 2.25\times10^{-3}W$$

$$P_{absorbed}(150 i_a) = -(V)(I_2) = -(150 i_a)\times I_2$$

$$= -(150\times -1\times10^{-3})\times1.5\times10^{-3}$$

$$= 2.25\times10^{-4}W$$

$P_{delivered}$ (2.5mA)



$V_{cu} \rightarrow$ Voltage across current source

$= +(V_{cu})I_1$

current I_1 is coming out of '+' terminal

KVL: $-V_{cu} + 400I_1 + 2.4\times10^3(I_1 - I_2) = 0$

$$V_{cu} = 400\times2.5\times10^{-3} + 2.4\times10^3(1\times10^{-3}) = 3.4V$$

$= +(3.4)(2.5\times10^{-3})$

$\boxed{P_{delivered} = 8.5\times10^{-3}W}$ (Total power delivered)

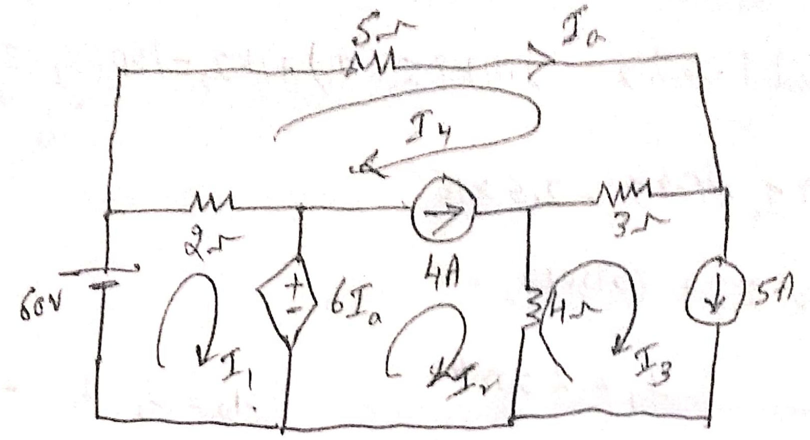
$P_{absorbed} = 2.5\times10^{-3} + 1.125\times10^{-3} + 2.4\times10^{-3} + 2.25\times10^{-3} + 2.25\times10^{-4} = \underline{8.5\times10^{-3}W}$

Total power absorbed

$\therefore$ Total power delivered = Total power absorbed

Q. For the circuit shown in figure, find the power supplied by 5A source and the output of the dependent source using mesh analysis.

Sol:



Sol:

control variable, $I_a = I_4$

KCL for non-essential mesh 3: $I_3 = 5$

KCL for supermesh 2 & 4: $I_2 - I_4 = 4$

KVL for essential mesh 1: $-60 + 2(I_1 - I_4) + 6\underbrace{I_4}_{I_a} = 0$

KVL for supermesh 2 & 4: $-6\underbrace{I_4}_{I_a} + 2(I_4 - I_1) + 5I_4 + 3(I_4 - \underbrace{5}_{I_3})$
$+ 4(I_2 - \underbrace{5}_{I_3}) = 0$

$$(0)I_1 + (1)I_2 + (-1)I_4 = 4$$
$$(2)I_1 + (0)I_2 + (4)I_4 = 60$$
$$(-2)I_1 + (4)I_2 + (4)I_4 = 35$$

$I_1 = 16.833A$ $\qquad I_3 = 5A$

$I_2 = 10.583A$

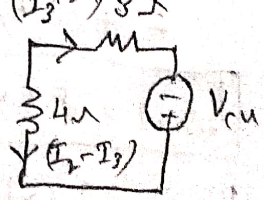
$I_4 = 6.58333A$

$P_{delivered}(5A) = +V_{cu} I_3$

$= +(-27.06)(5)$

$= -135.3\ W$

$P_{absorbed}(5A) = +135.3\ W$



$3(I_3 - I_4) - V_{cu} - 4(I_2 - I_3) = 0$

$V_{cu} = 3(I_3 - I_4) - 4(I_2 - I_3)$

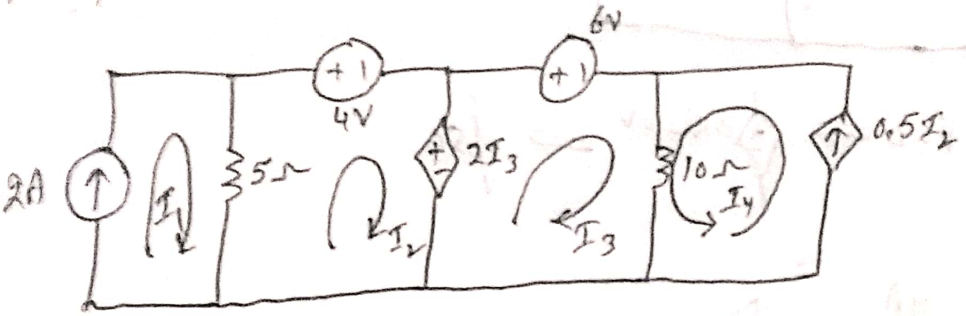
$V_{cu} = -27.06\ V$

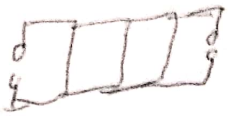
$$P_{absorbed}(6I_a) = -\underbrace{(6I_a)}_{\text{Voltage}} \times (I_2 - I_1)$$

$$= -6 \times 6.5833 \times (10.583 - 16.833)$$

$$= 246.87\,W$$

Q. Find the power delivered by the 6V source for the circuit shown in figure using loop analysis.





Sol: KCL to non-essential mesh 1 : $I_1 = +2$

KCL to non-essential mesh 4 : $I_4 = +0.5 I_2$

KVL to essential mesh 2 : $5(I_2 - \overset{I_1}{2}) + 4 + 2I_3 = 0$

KVL to essential mesh 3 : $-2I_3 + 6 + 10(I_3 + I_4) = 0$

$$(-0.5)I_2 + (0)I_3 + (1)I_4 = 0$$
$$(5)I_2 + (2)I_3 + (0)I_4 = 6$$
$$(0)I_2 + (8)I_3 + (10)I_4 = -6$$

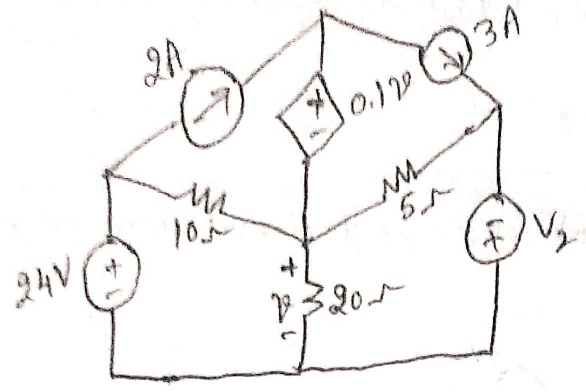
$I_2 = 2A$ $\qquad$ $I_1 = 2A$

$I_3 = -2A$

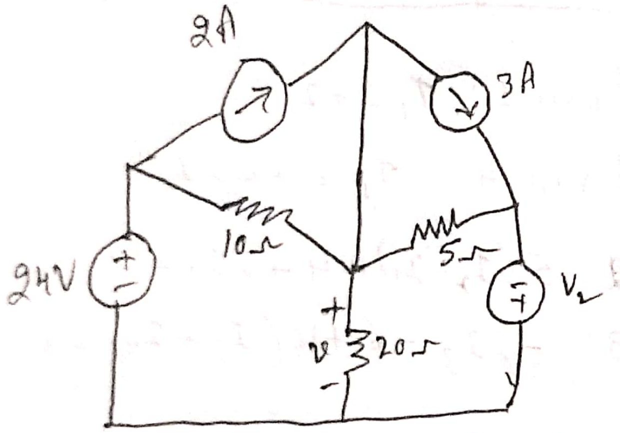
$I_4 = 1A$

$$P_{delivered(6V)} = -(V)(I_3)$$
$$= -(6)(-2)$$
$$= +12W$$

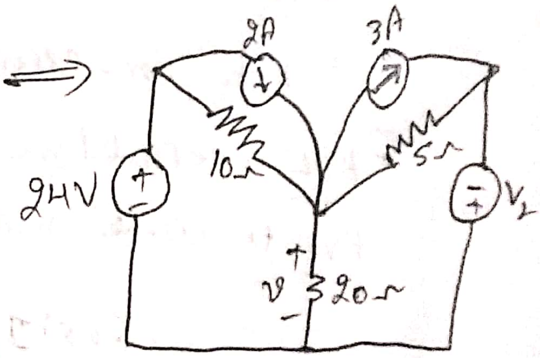
Q. Use mesh analysis to determine what value of V_2 in the network shown in figure causes $v=0$, v is the voltage across 20Ω.

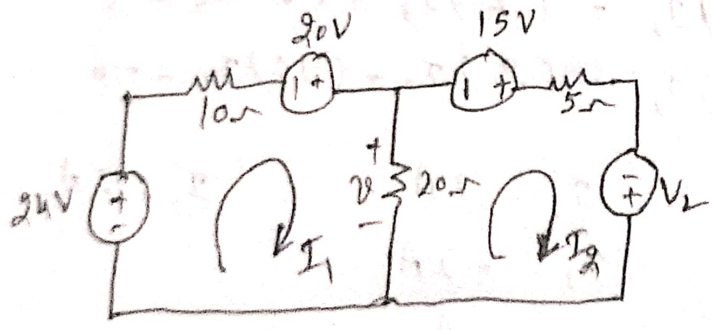


Given $v=0 \Rightarrow$ $0.1v = 0V$ $\Rightarrow$ s.c



$\Rightarrow$





$$v=0 \Rightarrow 20(I_1 - I_2) = 0 \Rightarrow I_1 = I_2$$

KVL to loop 1: $-24 + 10I_1 - 20 + 20\cancelto{0}{(I_1 - I_2)} = 0$

$$10I_1 = 44$$

$$I_1 = 4.4A \qquad I_2 = 4.4A$$

KVL to loop 2:

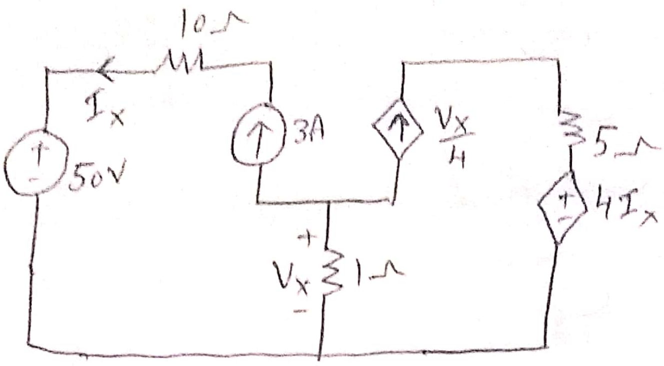
$$20\underbrace{\cancel{(I_2 - I_1)}}_{0} - 15 + 5I_2 - V_2 = 0$$

$$V_2 = -15 + 5I_2$$

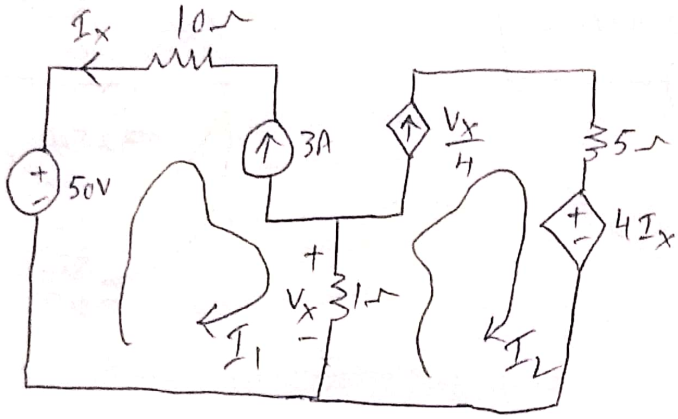
$$V_2 = -15 + 5 \times 4.4$$

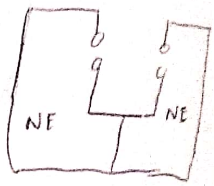
$$V_2 = 7V$$

1.c. Determine I_x and V_x for the circuit shown below using mesh analysis (7 marks)



Sol.





$$I_x = -I_1$$

$$V_x = +1(I_1 - I_2)$$

KCL equation for non essential mesh 1 : $I_1 = -3$ —①

KCL equation for non essential mesh 2 : $I_2 = \frac{+1(I_1 - I_2)}{4}$

$$\Downarrow$$

$$4I_2 = I_1 - I_2 \quad \text{—②}$$

Solving equations ① & ②

$$(1)I_1 + (0)I_2 = -3$$
$$(-1)I_1 + (5)I_2 = 0$$

$I_1 = -3A$ $\qquad$ $I_x = -I_1 = 3A$

$I_2 = -0.6\,A$ $\qquad$ $V_x = 1(I_1 - I_2) = 1(-3+0.6)$

$= -2.4V$